

National Curriculum of Pakistan
2022-23

MATHEMATICS

Grades 1-12



NATIONAL CURRICULUM COUNCIL SECRETARIAT
MINISTRY OF FEDERAL EDUCATION AND
PROFESSIONAL TRAINING, ISLAMABAD
GOVERNMENT OF PAKISTAN

- Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.

Progression Grid

Domain A: Numbers and Algebra

Number system is a system of representing numbers in mathematics. Students deal with various types of numbers for solving mathematics formulas and calculation, data processing and handling complex topics like algebra and geometry.

Algebra is one of the oldest branches in the history of mathematics that deals with number theory, geometry, and analysis. It is the study of mathematical symbols and the rules for manipulating these symbols in formulas; it is a unifying thread of almost all of mathematics. Algebra includes almost everything right from solving elementary equations to the study of abstractions. Also, there are several algebra equations, formulas and identities present in algebra.

Standard 1: The students will be able to:

- compare the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions.
- understand vectors and matrices as systems that have some of the properties of the real number system.

- use number-theory arguments to justify relationships involving whole number
- analyse and interpret mathematical situations by manipulating algebraic expressions and relations,
- model and solve contextual problems,
- interpret functions, calculate rate of change of functions, apply differentiation, integrate analytically,
- utilise integration, solve simple ordinary differential equations, solve nonlinear equations numerically by simple iterative formula.

Domain A: Numbers and Algebra

Grade 9	Grade 10	Grade 11	Grade 12
<p>Benchmark I: Students will be able to identify Real Numbers and their properties to carry out basic operations.</p> <p>Benchmark II: Students will be able to add, subtract, and multiply matrices, evaluate the determinant of matrices to find the inverse of matrices, solve simultaneous linear equations using matrices,</p> <p>Benchmark III: Students will be able to use Venn diagrams to demonstrate and describe operations of sets</p>		<p>Benchmark I: Students will be able to Identify complex numbers and their properties to carry out basic operations.</p> <p>Benchmark II: Students will be able to perform matrix algebra, evaluate determinant and solve homogeneous and non-homogeneous linear equations.,</p> <p>Benchmark III: Students will be able to demonstrate Arithmetic, geometric and harmonic sequence, their means and sum of series and apply them in real world problems.</p>	

and apply in real life situations. Express functions, inverse functions, and composite functions

Benchmark IV: Students will be able to simplify, factorise and manipulate Algebraic Fractions, Identify and rationalise surds, and factorise algebraic expressions

Benchmark V: Students will be able to solve linear equations, a system of two linear equations with two variables and solve linear inequalities.

Benchmark VI: Students will be able to solve Quadratic equations by using different methods and solve real world situations by formulating a quadratic equation

Benchmark VII: Students will be able to plot and interpret the Graphs in practical situations such as travel graphs, conversion graphs and speed time graphs.

Benchmark IV: Students will be able to apply the principle of Mathematical Induction to prove statements, identities, and formulae, and find approximate values of the binomial expansions having indices as rational numbers.

Benchmark V: Students will be able to divide polynomials, apply factor theorem, remainder theorem, factorise cubic polynomial and resolve an algebraic fraction into partial fractions

Benchmark VI: Students will be able to analyse attributes of quadratic equations and solve quadratic equations, and quadratic inequalities, in one unknown

Benchmark VII: Students will be able to plot and interpret the Graphs of functions. Fundamental transcendental functions, their domain and range. Evaluate limits of different algebraic, exponential, and trigonometric functions

Benchmark VIII: Students will be able to differentiate and integrate a function with the emphasis on practical applications.

Benchmark IX: Students will be able to find Solution of differential equations and apply first-order separable ordinary differential equations of degree one.

Benchmark X: Students will be able to solve nonlinear equations in one variable and definite integral by numerical methods.

Student Learning Outcomes

<p>Real Numbers</p> <ul style="list-style-type: none"> ● [SLO: M-09-A-01]: Explain, with examples, that civilizations throughout history have systematically studied living things [e.g., the history of numbers from Sumerians and its development to the present Arabic system. ● [SLO: M-09-A-02]: Describe the set of real numbers as a combination of rational and irrational numbers 	<p>Complex Number</p> <ul style="list-style-type: none"> ● [SLO: M-10-A-01]: Identify complex numbers, complex conjugate, absolute value or modulus of a complex number ● [SLO: M-10-A-02]: Apply algebraic properties and perform basic operations on complex numbers ● [SLO: M-10-A-03]: Demonstrate additive identity and multiplicative identity for the set of complex numbers ● [SLO: M-10-A-04]: Find additive inverse and multiplicative 	<p>Complex Number</p> <ul style="list-style-type: none"> ● [SLO: M-11-A-01] Recall complex number z represented by an expression of the form $z = a + ib$ or of the form (a, b) where a and b are real numbers and $i = \sqrt{-1}$ ● [SLO: M-11-A-02] Recognize a as a real part of z and b as an imaginary part of z. ● [SLO: M-11-A-03]: Know the condition for equality of complex numbers. ● [SLO: M-11-A-04]: Carry out basic operations on complex numbers 	
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- [SLO: M-09-A-03]: Demonstrate and verify the properties of equality and inequality of real numbers
- [SLO: M-09-A-04]: Apply laws of indices to simplify radical expressions
- [SLO: M-09-A-05]: Express a number in scientific notations and vice versa.
- [SLO: M-09-A-06]: Describe logarithm of a number
- [SLO: M-09-A-07]: Differentiate between common and natural logarithm

inverse of a complex number z .

- [SLO: M-10-A-05]: Demonstrate the following properties of a complex number z .

$$|z| = |-z| = |\bar{z}| \quad \left| \frac{z}{\bar{z}} \right| = \frac{|z|}{|z|} = 1$$

$$\overline{\overline{z}} = z, \quad z \bar{z} = |z|^2$$

$$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2, \quad \overline{\left(\frac{z_1}{z_2} \right)} = \frac{\bar{z}_1}{\bar{z}_2}, \quad z_2 \neq 0.$$

- [SLO: M-10-A-06]: Find real and imaginary parts of complex numbers of the type $(x+iy)^n$, $\left[\frac{x_1+iy_1}{x_2+iy_2} \right]^n$,

$$x_2 + iy_2 \neq 0$$

Where $n = \pm 1$ and ± 2

- [SLO: M-10-A-07]: Explain, with

- [SLO: M-11-A-05]: Define $z = a - ib$ as the complex conjugate of $z = a + ib$.
- [SLO: M-11-A-06]: Define $z = a^2 + b^2$ as the absolute value or modulus of a complex number $z = a + ib$

Solution of equations

- [SLO: M-11-A-07] i) Solve the simultaneous linear equations with

- [SLO: M-09-A-08]:
Apply laws of logarithm to real life situations such as growth and decay, loudness of sound.

- [SLO: M-09-A-09]:
Apply concepts of rational numbers to real word problems (such as inventory (stock taking), temperature, banking, measures of gain and loss, sources of income and expenditure).

examples, how mathematical models and equations are often used to make predictions and test hypotheses in science. [e.g, In physics, mathematical equations are used to describe the motion of objects and the behaviour of energy and matter. In chemistry, mathematical models are used to predict the behaviour of chemical reactions and the properties of molecules. In biology, mathematical models are used to predict the growth and spread of populations

complex coefficients. For example,

$$\begin{cases} 5z - (3+i)w = 7-i, \\ (2-i)z + 2iw = -1+i. \end{cases}$$

- [SLO: M-11-A-08]:
Write the polynomial $P(z)$ as a product of linear factors. For example,

$$\begin{aligned} z^2 + a^2 &= (z + ia)(z - ia) \\ z^3 - 3z^2 + z + 5 &= \\ (z + 1)(z - 2 - i)(z - 2 + i) \end{aligned}$$

- [SLO: M-11-A-09]:
Solve quadratic equation of the form

$2pz + qz + r = 0$
by completing squares, where p, q, r are real numbers and z a complex number. For example

Solve:
 $z^2 - 2z + 5 = 0$

	<p>and the spread of disease.]</p> <ul style="list-style-type: none"> ● SLO: M-10-A-08]: Solve the simultaneous linear equations with complex coefficients, ● [SLO: M-10-A-09]: Apply the Geometric interpretation of a complex number ● [SLO: M-10-A-10]: Apply the geometric interpretation of the modulus of a complex number. ● [SLO: M-10-A-11]: Apply the geometric interpretation of algebraic operations. ● [SLO: M-10-A-12]: Solve quadratic equations of the form ax^2 	$(z - 1 - 2i)(z - 1 + 2i) = 0$ $z = 1 + 2i, 1 - 2i$ <ul style="list-style-type: none"> ● [SLO: M-11-A-10]: Explain the polar coordinates system. ● [SLO: M-11-A-11]: Describe the polar representation of a complex number. ● [SLO: M-11-A-12]: Apply the operations with complex numbers in polar representation. ● [SLO: M-11-A-13]: Demonstrate simple equations and in-equations involving complex numbers in polar form. ● [SLO: M-11-A-14]: Apply concepts of complex numbers to real world problems (such as cryptography, wave phenomena, 	
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	<p>a is not equal to 0 by factorization, quadratic formula, completing square and graphs.</p> <ul style="list-style-type: none">• [SLO: M-10-A-13]: Draw the graphs of the quadratic function. $y = a^2$ a is not equal to 0• [SLO: M-10-A-14]: Establish relationship between roots and coefficients of quadratic equations.• [SLO: M-10-A-15]: Form a quadratic equation when roots are given.• [SLO: M-10-A-16]: Find discriminant of a given quadratic equation.• [SLO: M-10-A-17]: Identify the nature of	<p>calculate voltage, current, circuits, the velocity and pressure of the fluid).</p>	
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	<p>roots of a quadratic equation through discriminant.</p> <ul style="list-style-type: none">• [SLO: M-10-A-18]: Solve a pair of linear and quadratic equations simultaneously.• [SLO: M-10-A-19]: Solve word problems involving quadratic equations.• [SLO: M-10-A-20]: Solve quadratic inequalities in one unknown.• [SLO: M-10-A-21]: Apply the concept of quadratic equations, and quadratic inequalities, to real world problems (such as in physics, engineering, and finance, i.e. calculating max and		
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	<p>min heights in projectile motion, determining the max price on a company's budget, stability of population, growth of business, the relationship between hours worked and amount earned etc.).</p>		
	<p>Matrices and determinants</p> <ul style="list-style-type: none"> • [SLO: M-10-A-22]: Display information in the form of matrix of order 2. • [SLO: M-10-A-23]: Solve situations involving sum, difference, and product of two matrices 	<p>Matrices & Determinants</p> <ul style="list-style-type: none"> • [SLO: M-11-A-15]: Apply matrix operations (addition/subtraction and multiplication of matrices) with real and complex entries. • [SLO: M-11-A-16]: Evaluate determinants of 3×3 matrix by using cofactors and 	

	<ul style="list-style-type: none"> ● [SLO: M-10-A-24]: Calculate the product of the scalar quantity and a matrix ● [SLO: M-10-A-25]: Evaluate the determinant and inverse of a matrix of order 2×2. ● [SLO: M-10-A-26]: Solve the simultaneous linear equations in two variables using matrix inversion method and Cramer's rule ● [SLO: M-10-A-27]: Explain, with examples, how mathematics plays a key role in the development of new scientific theories 	<p>properties of determinants.</p> <ul style="list-style-type: none"> ● [SLO: M-11-A-17]: Use row operations to find the inverse and the rank of a matrix. ● [SLO: M-11-A-18]: Explain a consistent and inconsistent system of linear equations and demonstrate through examples ● [SLO: M-11-A-19]: Solve a system of 3 by 3 nonhomogeneous linear equations by using matrix inversion method and Cramer's Rule. ● [SLO: M-11-A-20]: Solve a system of three homogeneous linear equations in three unknowns using the 	
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	<p>and technologies. [e.g., Mathematical models and simulations are used to design and optimize new materials and drugs, and to understand the behaviour of complex systems such as the human brain.]</p> <ul style="list-style-type: none">● [SLO: M-10-A-28]: Apply concepts of matrices to real world problems (such as engineering, economics, computer graphics, and physics).	<p>Gaussian elimination method.</p> <ul style="list-style-type: none">● [SLO: M-11-A-21]: Apply concepts of matrices to real world problems such as (graphic design, data encryption, seismic analysis, cryptography, transformation of geometric shapes, social network analysis).	
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<p>Sets and Functions</p> <ul style="list-style-type: none"> • [SLO: M-09-A-10]: Describe mathematics as the study of pattern, structure, and relationships. • [SLO: M-09-A-11]: Identify sets and apply operations on three sets (Subsets, overlapping sets and disjoint sets), using Venn diagrams. • [SLO: M-09-A-12]: Solve problems on classification and cataloging by using Venn diagrams for Scenarios involving two sets and three sets. Further application of sets • [SLO: M-09-A-13]: Verify and apply 	<p>Functions and Graphs</p> <ul style="list-style-type: none"> • [SLO: M-10-A-29]: Recognize notation and determine the value of a function. • [SLO: M-10-A-30]: Identify types of functions (into, onto, one-to-one, injective, surjective and bijective) by using Venn diagrams. • [SLO: M-10-A-31]: Explain operations on, and compositions of, functions. • [SLO: M-10-A-32]: Find the inverse of a given function. • [SLO: M-10-A-33]: Formulate composite functions as defined by $gf(x) = f(g(x)).$ 	<p>Sequences and Series</p>	<p>Functions and Graphs</p> <ul style="list-style-type: none"> • [SLO: M-12-A-01]: Recall <ul style="list-style-type: none"> • function as a rule of correspondence, • domain, co-domain and range of a function, • one to one and onto functions. • [SLO: M-12-A-02]: Know linear, quadratic and square root functions. Inverse Function • [SLO: M-12-A-03]: Define inverse functions and demonstrate their domain and range with examples. <p>Graphical Representation of Functions</p> <ul style="list-style-type: none"> • [SLO: M-12-A-04]: Sketch graphs of
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<p>properties/laws of union and intersection of three sets through analytical and Venn diagram method</p> <ul style="list-style-type: none"> • [SLO: M-09-A-14]: Apply concepts from set theory to real world problems (such as in demographic classification, categorising products in shopping malls and music playlist by genre) Relation • [SLO: M-09-A-15]: Explain product, Binary Relations and its domain and range. • [SLO: M-09-A-16] Recognise that a relation can be represented by table, order pair and graphs. 	<ul style="list-style-type: none"> • [SLO: M-10-A-34]: Apply concepts from functions to real world problems (such as finance, transportation, and sales.) <p>Graphs of basic Functions</p> <ul style="list-style-type: none"> • [SLO: M-10-A-35]: Plot graphs of constant function, identity function, linear function and absolute valued functions • [SLO: M-10-A-36]: Solve absolute value equations and inequalities in one variable and express the solution as a range of values on a number line. 	<p>Arithmetic Sequence and Arithmetic Mean</p> <ul style="list-style-type: none"> • [SLO: M-11-A-22]: Solve problems by analysing arithmetic sequences and series up to n terms. <p>Geometric Sequence and Mean</p> <ul style="list-style-type: none"> • [SLO: M-11-A-23]: Solve problems by analysing geometric sequences and series up to n terms. • [SLO: M-11-A-24]: Identify a sequence as arithmetic or geometric sequence up to n terms. • [SLO: M-11-A-25]: Solve problems by analysing harmonic sequences and series up to n terms. 	<ul style="list-style-type: none"> • linear functions (e.g. $y = ax + b$), • non-linear functions (e.g. $2y = x$). • [SLO: M-12-A-05]: Plot the graph of the function $y = x^n$ here n is <ul style="list-style-type: none"> • a + ve integer, • a - ve integer ($x \neq 0$), • a rational number for $x > 0$ • [SLO: M-12-A-06]: Plot graph of quadratic function of the form $y = ax^2 + bx + c$ $a(\neq 0)$, b, c are integers. • [SLO: M-12-A-07]: Draw graph using factors.
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	<ul style="list-style-type: none"> • [SLO: M-10-A-37]: Apply concepts of absolute valued functions to real-world problems (such as to calculate energy wave, magnitude and distance). • [SLO: M-10-A-38]: Apply concepts from functions to real world problems (such as finance, transportation, and sales.) 	<p>Miscellaneous Series</p> <ul style="list-style-type: none"> • [SLO: M-11-A-26]: Find sum of: <ul style="list-style-type: none"> • the first n natural numbers ($\sum n$), • the squares of the first n natural numbers ($\sum n^2$), • the cubes of the first n natural numbers ($\sum n^3$). • [SLO: M-11-A-27]: Recognize the arithmetic geometric sequence, determine its general term, find sum to n terms and sum to infinite number of terms, using sigma notation. • [SLO: M-11-A-28]: Identify leasing of 	<ul style="list-style-type: none"> • [SLO: M-12-A-08]: Predict functions from their graphs (use the factor form to predict the equation of a function of the type $f(x) = a(x - p)(x - q)$ (if two points where the graph crosses <i>x axis</i> and third point on the curve, are given). Intersecting Graphs • [SLO: M-12-A-09]: Find the intersecting point graphically when intersection occurs between <ul style="list-style-type: none"> • a linear function and coordinate axes, • two linear functions, • a linear and a quadratic function.
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		<p>motor vehicles, down payment, motor vehicle insurance, processing charges, repayment in monthly instalments.</p> <ul style="list-style-type: none"> • [SLO: M-11-A-29]: Solve problems related to leasing of motor vehicle under different conditions. • [SLO: M-11-A-30]: Apply concepts from sequence and series to real world problems (such as simple interest on loan, investment, depreciation, Investment planning on compound interest, projectile motion, gaming strategy, health care management, web 	<ul style="list-style-type: none"> • [SLO: M-12-A-10]: Solve, graphically, appropriate problems from daily life. • [SLO: M-12-A-11]: Classify the functions as algebraic and transcendental functions • [SLO: M-12-A-12]: Describe various transcendental functions, such as: <ul style="list-style-type: none"> • Trigonometric functions, • Inverse trigonometric functions, • Logarithmic function, • Exponential function <p>Logarithmic function</p>
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		<p>page design, traffic modelling).</p>	<ul style="list-style-type: none"> • [SLO: M-12-A-13]: Demonstrate an understanding of logarithms. • [SLO:M-12-A-14]: Derive and apply product, quotient, and power laws of logarithms <p>Exponential function</p> <ul style="list-style-type: none"> • [SLO: M-12-A-15]: Graph and analyse exponential and logarithmic function Apply the concept of exponential function to find compound interest • [SLO: M-12-A-16]: Solve problems that involve exponential and logarithmic Equations • [SLO: M-12-A-17]: Identify the domain
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			<p>and range of fundamental transcendental functions. Graphical Representation</p> <ul style="list-style-type: none">• [SLO: M-12-A-18]: Draw the graph of modulus function• [SLO: M-12-A-19]: Interpret the relation between a one-one function and its inverse through a graph.• [SLO: M-12-A-20]: Demonstrate the transformations of a graph through horizontal shift, vertical shift, and scaling. <p>Limit of a Function</p> <ul style="list-style-type: none">• [SLO: M-12-A-21]: Demonstrate and find the limit of a
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			<p>sequence and a function.</p> <ul style="list-style-type: none">• [SLO: M-12-A-22]: State and apply theorems on limit of sum, difference, product and quotient of functions to algebraic, exponential and trigonometric functions. <p>Continuous and Discontinuous Functions</p> <ul style="list-style-type: none">• [SLO: M-12-A-23]: Demonstrate and test Continuity, discontinuity of a function at a point and in an interval.• [SLO: M-12-A-24]: Apply concepts of transcendental functions, limit of a function and its
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			<p>continuity to real world problems such as (growth and decay, finance, economics, surveying, navigation, astronomy, growth rate of sales, rate of change in sales, predicting long-term stock prices).</p> <ul style="list-style-type: none">• [SLO: M-12-A-25]: Calculate inflation over a period of time.• [SLO: M-12-A-26]: Calculate depreciation with the help of straight-line method, sum of years digit method and production unit method
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		<p>Mathematical Induction</p> <ul style="list-style-type: none"> • [SLO: M-11-A-31]: Describe a mathematical argument, identify the base case, induction of hypothesis and a precise conclusion. • [SLO: M-11-A-32]: Apply the principle of mathematical induction to prove statements, identities, divisibility of numbers and summation formulae. • [SLO: M-11-A-33]: Evaluate and justify conclusions, communicating a position clearly in an appropriate mathematical form in daily life. <p>Binomial Theorem</p>	<p>Gradient of a curve</p> <ul style="list-style-type: none"> • [SLO: M-12-A-27]: Recognize the meaning of the tangent to a curve at a point. • [SLO: M-12-A-28]: Calculate the gradient of a curve at a point. • [SLO: M-12-A-29]: Identify the derivative as the limit of a difference quotient. • [SLO: M-12-A-30]: Calculate the derivative of a given function at a point. • [SLO: M-12-A-31]: Estimate the derivative as rate of change of velocity,
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		<ul style="list-style-type: none"> • [SLO: M-11-A-34]: State and apply the Binomial Theorem to expand expressions of the form $(a + b)^n$ where n is a positive integer. • [SLO: M-11-A-35]: Describe Binomial Theorem as expansion of binomial powers restricted to the set of natural numbers. • [SLO: M-11-A-36]: Calculate binomial coefficients using Pascal's triangle. • [SLO: M-11-A-37]: Expand using the binomial theorems, and use appropriate techniques to simplify the expression • [SLO: M-11-A-38]: Find an approximate value using binomial 	<p>temperature and profit.</p> <p>Derivative of a Function</p> <ul style="list-style-type: none"> • [SLO: M-12-A-32]: Recognize the derivative function. • [SLO: M-12-A-33]: Find the derivative of a square root function. • [SLO: M-12-A-34]: Find the derivative of a quadratic function. • [SLO: M-12-A-35]: State the connection between derivatives and continuity. <p>Differentiation Rules</p> <ul style="list-style-type: none"> • [SLO: M-12-A-36]: State, prove and apply the constant rule, the coefficient rule, the
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		<p>theorem Applications of Binomial Theorem</p> <ul style="list-style-type: none"> • [SLO: M-11-A-39]: Use binomial theorem to find the remainder when a number to some large exponent is divided by a number. • [SLO: M-11-A-40]: Use binomial theorem to find the last digit of a number, test the divisibility by a number and compare two large numbers. • [SLO: M-11-A-41]: Apply concepts of Mathematical induction and binomial theorem to real world problems such as (puzzles, domino effects, Pascal's triangle, Economic forecasting, 	<p>power rule, the sum and difference rule, the product rule, the quotient rule for differentiation.</p> <ul style="list-style-type: none"> • [SLO: M-12-A-37]: Extend the power rule to functions with negative exponents. • [SLO: M-12-A-38]: Combine the differentiation rules to find the derivative of a polynomial or rational function. • [SLO: M-12-A-39]: Apply rates of change to displacement, velocity and acceleration of an object moving along a straight line. <p>Further on Differentiation</p> <ul style="list-style-type: none"> • [SLO: M-12-A-40]:
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		<p>Rankings, Variable subletting)</p>	<p>Find the derivative of trigonometric and inverse trigonometric functions.</p> <ul style="list-style-type: none"> • [SLO: M-12-A-41]: <p>Find the derivative of exponential functions.</p> <ul style="list-style-type: none"> • [SLO: M-12-A-42]: <p>Find the derivative of logarithmic functions.</p> <ul style="list-style-type: none"> • [SLO: M-12-A-43]: <p>Apply differentiation to state the increasing and decreasing functions.</p> <ul style="list-style-type: none"> • [SLO: M-12-A-44]: <p>Apply differentiation to find equations of tangent and normal to a curve at a point.</p> <ul style="list-style-type: none"> • [SLO: M-12-A-45]: <p>Apply concepts of Differentiation to real</p>
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			<p>world problems such as (profits on diminishing returns, environmental factors, financial investments, population growth, spread of diseases, movement of particles, time-speed in transportation, structural stress, material required viz a viz changes in construction).</p> <p>Higher-Order Derivatives</p> <ul style="list-style-type: none"> • [SLO: M-12-A-46]: <p>Find higher order derivatives of algebraic, implicit, parametric, trigonometric, inverse-trigonometric, exponential and logarithmic functions.</p> <p>Applications of Derivatives</p> <ul style="list-style-type: none"> • [SLO: M-12-A-47]: <p>Describe the ability to approximate functions</p>
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			<p>locally by linear functions. (Linear approximations of square root functions, trigonometric functions)</p> <ul style="list-style-type: none"> • [SLO: M-12-A-48]: Explain differentials and draw a graph that illustrates the use of differentials to approximate the change in a quantity. • [SLO: M-12-A-49]: Calculate the relative error and percentage error in using a differential approximation. (Volume of a cube and sphere) <p>Extreme Values</p> <ul style="list-style-type: none"> • [SLO: M-12-A-50]: Illustrate Global extrema (absolute extrema) and local
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			<p>extrema (relative extrema)</p> <ul style="list-style-type: none">• [SLO: M-12-A-51]: Find the extreme values by applying the second derivative test.• [SLO: M-12-A-52]: Explain how to find the critical points of a function over a closed interval.• [SLO: M-12-A-53]: Describe how to use critical points to locate absolute extrema over a closed interval.• [SLO: M-12-A-54]: Apply derivatives to real-world problems to find the maximum and the minimum values of a function
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			<p>under certain conditions.</p> <ul style="list-style-type: none"> • [SLO: M-12-A-55]: Apply the concept of higher order derivatives to real life problems (such as transportation devices, cars, planes, roller coasters, rate of spread of a disease, rate of improvement of performance in psychology, automobiles, radar guns, economics).
			<p>Integration I</p> <ul style="list-style-type: none"> • [SLO: M-12-A-56]: Find the general antiderivative of a given function. • [SLO: M-12-A-57]: Recognize and use the terms and

			<p>notations for antiderivatives.</p> <ul style="list-style-type: none">● [SLO: M-12-A-58]: State the power rule for integrals.● [SLO: M-12-A-59]: State and apply the properties of indefinite integrals.● [SLO: M-12-A-60]: State the definition of the definite integral.● [SLO: M-12-A-61]: Explain the terms integrand, limits of integration, and variable of integration.● [SLO: M-12-A-62]: State and apply the properties of definite integrals.● [SLO: M-12-A-63]: State and apply Fundamental Theorem of Calculus
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			<p>to evaluate the definite integrals.</p> <ul style="list-style-type: none">● [SLO: M-12-A-64]: Describe the relationship between the definite integral and net area.● [SLO: M-12-A-65]: Find the area of a region bounded by a curve and lines parallel to axes, or between a curve and a line, or between two curves.● [SLO: M-12-A-66]: Find Volume of revolution about one of the axes.● [SLO: M-12-A-67]: Demonstrate trapezium rule to estimate the value of a definite integral.● [SLO: M-12-A-68]: Apply concepts of
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			Integration to real world problems such as (volume of a container, consumer surplus and producer surplus, growth rate of a population, investment return time period, drug dosage required by integrating the concentration).
<p>Factorization:</p> <ul style="list-style-type: none"> • [SLO: M-09-A-17]: Identify common factors, trinomial factoring, concretely, pictorially and symbolically. • [SLO: M-09-A-18]: Factorize quadratic and cubic algebraic expressions: <ul style="list-style-type: none"> • $a^4 + a^2b^2 + b^4$ or $a^4 + a^2b^2 + b^4$ 	<p>Algebraic Fractions:</p> <ul style="list-style-type: none"> • [SLO: M-10-A-39]: Describe rational expressions • [SLO: M-10-A-40]: Factorize and simplify rational expressions. • [SLO: M-10-A-41]: Demonstrate manipulation of algebraic fractions. 	<p>Division of polynomial:</p> <ul style="list-style-type: none"> • [SLO: M-11-A-42]: Divide a polynomial of degree up to 4 by a linear and quadratic polynomial to identify quotient and remainder. Remainder Theorem and Factor Theorem: • [SLO: M-11-A-43]: Demonstrate and apply remainder theorem 	

<ul style="list-style-type: none"> • $x^4+px + q$ • $ax^2+bx + c$ • $(ax^2+bx + c)(ax^2 + bx + d)$ • $(x + a)(x + b)(x + c)$ • $(x + a)(x + b)(x + c)(x + d)$ • $a^3+3a^2b+3ab^2+b^3$ • $a^3-3a^2b+3ab^2-b^3$ • $a^3\pm b^3$ <ul style="list-style-type: none"> • [SLO: M-09-A-19]: Find highest common factor and least common multiple of algebraic expressions and know relationship of LCM and HCF • [SLO: M-09-A-20]: Find square root of algebraic expression by factorization and division • [SLO: M-09-A-21]: Apply the concepts of factorization of 	<ul style="list-style-type: none"> • [SLO: M-10-A-42]: Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). • [SLO: M-10-A-43]: Apply the concept of rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials) to real world problems (such as the amount of work a person can do in certain amount of time, rates, and work) 	<ul style="list-style-type: none"> • [SLO: M-11-A-44]: Analyse and apply factor theorem to factorise a cubic polynomial • [SLO: M-11-A-45]: Apply concepts of remainder and factor theorem to real world problems (such as polynomial regression, signal processing, and coding theory). 	
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<p>quadratic and cubic algebraic expressions to real world problems (such as engineering, physics, and finance.)</p>			
<p>Linear Equations and Inequalities in one variable:</p> <ul style="list-style-type: none"> • [SLO: M-09-A-22]: Solve linear equations and inequalities with rational coefficients and represent the solution set on a real line • [SLO: M-10-A-23]: Solve two linear inequalities with two unknowns simultaneously 	<p>Linear Inequalities in two variables:</p> <ul style="list-style-type: none"> • [SLO: M-10-A-44]: Solve two linear inequalities with two unknowns simultaneously • [SLO: M-10-A-45]: Interpret and Identify regions in plane bounded by two linear inequalities in two unknowns. <p>Quadratic Equations</p> <ul style="list-style-type: none"> • [SLO: M-10-A-46]: Solve quadratic 		

	<p>equations by using the methods of:</p> <ul style="list-style-type: none"> • factorization, • completing squares, and • quadratic formula. <ul style="list-style-type: none"> • [SLO: M-10-A-47]: Solve problems of “changing the subject of formula”. • [SLO: M-10-A-48]: Solve fractional equations that can be reduced to quadratic equations. • [SLO: M-10-A-49]: Solve real world situations by formulating a quadratic equation 		
	<p>Plotting and Interpreting the Graphs</p>		<p>Mechanics Kinematics of motion in a straight line</p>

- [SLO: M-10-A-50]: Draw graphs of functions of the form $y = ax$ (Including the sums of few of these and taking n as a rational number)
- [SLO: M-10-A-51]: Solve a system of one linear and one quadratic equation graphically and interpret the solution.
- [SLO: M-10-A-52]: Discover exponential growth/decay of a practical phenomenon through its graph.
- [SLO: M-10-A-53]: Determine the gradients of curves through tangents.
Curves sketching

- [SLO: M-12-A-69]: Recognise distance and speed as scalar quantities, and displacement, velocity and acceleration as vector quantities.
- [SLO: M-12-A-70]: Sketch and interpret displacement–time graphs and velocity–time graphs
- [SLO: M-12-A-71]: use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration
- [SLO: M-12-A-72]: use appropriate formulae for motion with constant

- [SLO: M-10-A-54]: Identify, sketch and interpret graphs of the Linear functions
- [SLO: M-10-A-55]: Identify, sketch and interpret graphs of the Nonlinear functions such as Quadratic, Cubic, Reciprocal, and Exponential.
- [SLO: M-10-A-56]: Sketch graph of the function $y = x^n$ where n is a + ve integer, - ve integer, rational number for $x > 0$
- [SLO: M-10-A-57]: Apply concepts of sketching and interpreting graph to real life problems (such as in tax payment, income

acceleration in a straight line

- [SLO: M-12-A-73]: Apply the concept of mechanics to real life problems (such as motion of vehicles on roads, Projectile motion, free fall motion, relative motion animation)

Integration II Techniques of Integration

- [SLO: M-12-A-74]: Utilise trigonometric relationships to evaluate integrals.
- [SLO: M-12-A-75]: Integrate functions involving the exponential and logarithmic functions.
- [SLO: M-12-A-76]: Identify when to use

	and salary problems and cost and profit analysis)		<p>integration by parts to solve integration problems.</p> <ul style="list-style-type: none">• [SLO: M-12-A-77]: Apply the integration-by-parts formula for definite integrals.• [SLO: M-12-A-78]: Solve integration problems involving trigonometric substitution• [SLO: M-12-A-79]: Integrate a rational function using the method of partial fractions.• [SLO: M-12-A-80]: Apply concepts of integration to real life word problems (such as area between curves, average value of a function, distance velocity,
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			<p>acceleration, growth rate of population, moment of inertia, vector calculus to measure sensor network).</p>
			<p>Differential Equations</p> <ul style="list-style-type: none"> • [SLO: M-12-A-81]: Identify and construct first order differential equations from practical situations. Solution of Differential Equation • [SLO: M-12-A-82]: Solve separable differential equations of first order and first degree of y separable variable equations y Homogeneous equations

			<ul style="list-style-type: none"> • [SLO: M-12-A-83]: Apply concepts of first order differential equations to real life word problems (such as population growth and decay, Cooling/Warming law, flow of electricity, series circuits, economics and finance, radioactive decay).
			<p>Numerical Solution of Nonlinear Equations</p> <ul style="list-style-type: none"> • [SLO: M-12-A-84]: Analyse the searching of roots of an equation by graphical means and/or searching for the sign change.

			<ul style="list-style-type: none">• [SLO: M-12-A-85]: Explain the basic principles of solving a nonlinear equation in one variable.• [SLO: M-12-A-86]: Calculate real roots of a nonlinear equation in one variable by bisection method, regular-falsi method, Newton Raphson method.• [SLO: M-12-A-87]: Apply concepts of nonlinear equations to real life word problems (such as chemical reactions, regulation of heart beats, electronic circuits, and cryptography).
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Domain B: Geometry

Geometry is the most practical branch of mathematics that deals with shapes and sizes of figures and their properties. The basic elements of geometry are points, lines, angles, surfaces and solids. Develops reasoning and problem-solving skills by applying properties of lines, triangles, quadrilaterals, and circles. length, perimeter, area, circumference, surface area, and volume to solve real-world problems.

Standard 2: The students will be able to:

- apply characteristics and properties of angles, triangles, parallelograms and circles to develop arguments about their geometric relationships.
- solve problems involving coordinate geometry, plane analytical geometry and vectors.

- recognize trigonometric identities, analyze conic sections, draw and interpret graphs of functions.

Grade 9	Grade 10	Grade 11	Grade 12
<p>Benchmark I: Students will be able to use and interpret Cartesian coordinates in two dimensions and solve problems involving coordinate geometry</p> <p>Benchmark II: Students will be able to Identify vectors in plane and apply vector addition, dot/ cross product, scalar product,</p> <p>Benchmark III: Students will be able to find volume and surface area of composite solids and solve problems using the relationship between areas of similar figures and volume of different solids</p> <p>Benchmark IV: Students will be able to apply characteristics and properties of angles, triangles, parallelograms and circles to develop arguments about their geometric relationships.</p> <p>Benchmark V: Students will be able to use trigonometric identities to verify relationships between trigonometric ratios. Apply appropriate laws and formulae of trigonometry to solve the triangles and relevant problems.</p> <p>Benchmark VI: Students will be able to calculate unknown angles and solve problems by using the properties of circles</p>			<p>Benchmark I: Students will be able to interpret and solve plane analytical geometry problem situations</p> <p>Benchmark II: Students will be able to Identify vectors in space and apply vector addition, dot/ cross product, scalar product, differentiate and integrate vector functions.</p> <p>Benchmark III: Students will be able to Identify and analyse conic sections (circle, parabola, ellipse and hyperbola) and solve related problems.</p> <p>Benchmark IV: Students will be able to apply trigonometric identities and formulas to solve relevant situations and draw graphs of trigonometric and inverse trigonometric functions.</p>

Student Learning Outcomes

Coordinate Geometry

- [SLO: M-09-B-01]: Derive distance formula by locating the position of two points in coordinate plane
- [SLO: M-09-B-02]: Calculate the midpoint of a line segment
- [SLO: M-09-B-03]: Find the gradient of a straight line when coordinates of two points are given
- [SLO: M-09-B-04]: Find the equation of a straight line in the form $y = mx + c$
- [SLO: M-09-B-05]: Find the gradient of

Analytical Geometry Concurrency of Straight lines

- [SLO: M-12-B -01]: Find the condition of concurrency of three straight lines.
- [SLO: M-12-B -02]: Find the equation of median, altitude and right bisector of a triangle.
- [SLO: M-12-B -03]: Show that
 - three right bisectors,
 - three medians,
 - three altitudes of a triangle are concurrent.

Area of a Triangular Region

<p>parallel and perpendicular lines</p> <ul style="list-style-type: none"> • [SLO: M-09-B-06]: Apply distance and midpoint formulas to solve real life situations such as physical measurements or distances between locations. • [SLO: M-09-B-07]: Apply concepts from coordinate Geometry to real world problems (such as, aviation and navigation, landscaping, map reading, longitude and latitude). • [SLO: M-09-B-08]: Derive equation of a straight line in slope-intercept form, 			<ul style="list-style-type: none"> • [SLO: M-12-B -04]: Find the area of a triangular region whose vertices are given. <p>Homogeneous Equations</p> <ul style="list-style-type: none"> • [SLO: M-12-B -05]: Recognize homogeneous linear and quadratic equations in two variables. • [SLO: M-12-B -06]: Investigate that the 2nd degree homogeneous equation in two variables \bar{z} and \bar{z} represents a pair of straight lines through the origin and finds an acute angle between them.
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point-slope form,
two-point form,
intercepts form,
symmetric form,
normal form.

- [SLO: M-09-B-09]:
Show that a linear
equation in two
variables represents a
straight line and
reduce the general
form of the equation
of a straight line to
the other standard
forms.

Angle Between Lines

- [SLO: M-09-B-10]:
Find the angle
between two coplanar
intersecting straight
lines.
- [SLO: M-09-B-11]:
Find the equation of
the family of lines

- [SLO: M-12 -B -07]:
Apply concepts of
analytical geometry
to real life world
problems (such as
aviation, to track
stars, distance
between planets and
satellites, space
science and
engineering).

passing through the point of intersection of two given lines.

- [SLO: M-09-B-12]: Calculate angles of the triangle when the slopes of the sides are given.

Logic

- [SLO: M-09-B-13]: Differentiate between a mathematical statement and its proof.
- [SLO: M-09-B-14]: Differentiate between an axiom, conjecture and theorem.
- [SLO: M-09-B-15]: Formulate simple deductive proofs [algebraic proofs that require showing the LHS to be equal to

<p>the RHS. E.g., showing</p> $(x - 3)^2 + 5 = x^2 - 6x$			
	<p>Vectors in Plane</p> <ul style="list-style-type: none"> • [SLO: M-10-B-01]: Introduce rectangular coordinate system in plane. • [SLO: M-10-B-02]: Represent vectors as directed line segment • [SLO: M-10-B-03]: Express a vector in terms of two non-zero and non-parallel coplanar vectors. • [SLO: M-10-B-04]: Express a vector in terms of position vector • [SLO: M-10-B-05]: Express translation by a vector 	<p>Vectors in Space</p> <ul style="list-style-type: none"> • [SLO: M-11-B-01]: Recognize rectangular coordinate system in space. • [SLO: M-11-B-02]: Recognize: unit vectors \bar{i}, \bar{j} and \bar{k}. components of a vector. • [SLO: M-11-B-03]: Find the magnitude of a vector. • [SLO: M-11-B-04]: Repeat all fundamental mathematical operations for vectors in space which, in the plane, have already been discussed. • [SLO: M-11-B-05]: Demonstrate and prove 	<p>Vector Valued Function:</p> <ul style="list-style-type: none"> • [SLO: M-12-B -08]: Explain the need for a vector valued function. • [SLO: M-12-B -09]: Construct vector valued function. • [SLO: M-12-B-10]: Identify domain and range of vector valued functions. • [SLO: M-12-B -11]: Identify difference between scalar and vector valued functions Derivative of Vector Function • [SLO:M-12-B-12]: Explain derivative of a vector function of a

	<ul style="list-style-type: none"> • [SLO: M-10-B-06]: Find the magnitude of a vector. • [SLO: M-10-B-07]: Add and subtract vectors, • [SLO: M-10-B-08]: Multiply a vector by a scalar • [SLO: M-10-B-09]: Solve geometrical problems involving the use of vectors • [SLO: M-10-B-10]: Apply concepts from geometrical problems involving the use of vectors (such as parallel and perpendicular lines in geometrical shapes, vector projectile motion, crosswinds aviation, military 	<p>properties of Vector Addition</p> <ul style="list-style-type: none"> • Commutative law for vector addition. • Associative law for vector addition. • 0 as the identity for vector addition. • $-\vec{z}$ as the inverse for \vec{z}. • Dot or Scalar Product • [SLO: M-11-B-06]: Explain dot or scalar product of two vectors and give its geometrical interpretation. • [SLO: M-11-B-07]: Express dot product in terms of components. • [SLO: M-11-B-08]: Find the condition for orthogonality of two vectors. • [SLO: M-11-B-09]: Use dot product to find the angle between two vectors. 	<p>single variable and elaborate the result: If $f(t) = f_1(t)i + f_2(t)j + f_3(t)k$</p> <p>Where $f_1(t), f_2(t), f_3(t)$ are differentiable functions of a scalar variable t, then</p> <p>are differentiable functions of a scalar variable t, then</p> $\frac{df}{dt} = \frac{df_1}{dt}i + \frac{df_2}{dt}j + \frac{df_3}{dt}k$ <ul style="list-style-type: none"> • [SLO: M-12-B -13]: Apply vector differentiation to calculate velocity and acceleration of a position vector $f(t) = x(t)i + y(t)j$ • [SLO: M-12 -B -14] Apply concepts of vector valued
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	usage, designing roller coasters).	<ul style="list-style-type: none"> ● [SLO: M-11-B-10]: Find the projection of a vector along another vector. ● [SLO: M-11-B-11]: Find the work done by a constant force in moving an object along a given vector. ● [SLO: M-11-B-12]: Solve daily life problems based on vectors. Cross or Vector Product ● [SLO: M-11-B-13]: Explain the cross or vector product of two vectors and give its geometrical interpretation. ● [SLO: M-11-B-14]: Apply a cross product to find the angle between two vectors. ● [SLO: M-11-B-15]: Solve situations in daily 	functions to real life word problems (such as engineering and transportation).
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life based on Cross or dot
Vector Product.

Scalar Triple Product

- [SLO: M-11-B-16]: Describe scalar triple product of vectors.
- [SLO: M-11-B-17]: Express scalar triple product of vectors in terms of components (determinant form).
- [SLO: M-11-B-18]: Prove that:
 - $i \cdot j \times k = j \cdot i \times k$
 - $i \cdot k \times j = j \cdot i \times k$Prove that dot and cross are inter-changeable in scalar triple product.
- [SLO: M-11-B-19]: Find the volume of
 - a parallelepiped,
 - tetrahedron, determined by three given vectors.
- [SLO: M-11-B-20]: Define coplanar vectors

		<p>and find the condition for planarity of three vectors.</p> <ul style="list-style-type: none"> • [SLO: M-11-B-21]: Apply concepts of vectors in space to real world problems such as (design and execute optimal navigation paths in transportation and logistics, graphing complex 3D motion, vector operations in engineering and computer graphics, practical proficiency for work, flux, and circulation). 	
<p>Similar Figures</p> <ul style="list-style-type: none"> • [SLO: M-09-B-16]: Identify similarity of polygons. Area and Volume of Similar Figures • [SLO: M-09-B-17]: Solve problems using 			

the relationship between areas of similar figures and volume of different solids

Geometrical Properties of regular polygons, Triangles and Parallelograms

- [SLO: M-09-B-18]: Solve real life problems that involve the properties of regular polygons, triangles and parallelograms (such as building architectural structures, fencing, tiling, painting, carpeting a room).

Loci

<ul style="list-style-type: none"> [SLO: M-09-B-19]: Solve real life problems using the following loci and the method of intersecting loci for sets of points in two dimensions which are: at a given distance from a given point, at a given distance from a given straight line, equidistant from two given points equidistant from two given intersecting straight lines 			
<p>Trigonometry</p> <ul style="list-style-type: none"> [SLO: M-09-B-20]: Identify angles in standard position, expressed in degrees and radians 	<p>Application of Trigonometry</p> <ul style="list-style-type: none"> [SLO: M-10-B-11]: Extend sine and cosine functions to 	<p>Fundamental Law of Trigonometry</p> <ul style="list-style-type: none"> [SLO: M-11-B -22]: Establish fundamental law of trigonometry: 	<p>Inverse Trigonometric Functions</p> <ul style="list-style-type: none"> [SLO: M-12-B-15]: Find domains and ranges of principal trigonometric

- [SLO: M-09-B-21]: Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to find a side or of an angle of a right-angled triangle.
- [SLO: M-09-B-22]: Solve real life trigonometric problems in two dimensions involving angles of elevation and depression.

Trigonometric Identities

- [SLO: M-09-B-23]: Prove the trigonometric identities and apply them to show different trigonometric relations.

angles between 90° and 180°

- [SLO: M-10-B-12]: Solve problems using the laws of sine, cosine and the area formulas for any triangle.
- [SLO: M-10-B-13]: solve simple trigonometric problems in three dimensions
- [SLO: M-10-B-14]: Apply concepts of trigonometry to real life world problems (such as video games, flight engineering, navigation, sound waves)

- [SLO: M-11-B -23]: Apply fundamental law and its deductions to derive: Trigonometric ratios of allied angles, double angle, half angle and triple angle identities
- [SLO: M-11-B -24]: Express the product (of sines and cosines) as sums or differences (of sines and cosines).

Trigonometric Functions

- [SLO: M-11-B -25]: Find the domain and range of the trigonometric functions
- [SLO: M-11-B -26]: Discuss even, odd functions and the periodicity of trigonometric functions
- [SLO: M-11-B -27]: Find the maximum and minimum value of a

functions, inverse trigonometric functions. Graphs of Inverse Trigonometric Functions

- [SLO: M-12-B-16]: Draw the graphs of the inverse trigonometric functions of cosine, sine, tangent, secant, cosecant and cotangent within the domain from -2π to 2π .

Inverse trigonometric identities and solution of trigonometric equations

- [SLO: M-12-B-17]: State, prove and apply the addition and subtraction formulae of inverse

- [SLO: M-09-B-24]: Solve real life problems involving trigonometric identities.

Bearing

- [SLO: M-09-B-25]: Interpret and use three figure bearings.
- [SLO: M-09-B-26]: Solve problems involving bearing.
- [SLO: M-09-B-27]: Apply the concepts of trigonometry

given function of the type:

- $a + b\sin\theta$,
- $a + b\cos\theta$
- $a + b\sin(c\theta + d)$,
- $a + b\cos(c\theta + d)$
- the reciprocals of above, where a, b, c and d are real numbers.

Graphs of Trigonometric Functions

- [SLO: M-11-B-28]: Graph and analyse the trigonometric functions sine, cosine, and tangent to solve problems,
- [SLO: M-11-B -29]: Explain the properties of graphs of $\sin\theta$, $\cos\theta$ and $\tan\theta$.
- [SLO: M-11-B -30]: Apply the concepts of trigonometric functions, identities, graphs,

trigonometric functions

- [SLO: M-12-B-18]: Solve trigonometric equations of the type $\sin\theta = k$, $\cos\theta = k$ and $\tan\theta = k$,

using periodic, even/odd and translation properties.

- [SLO: M-12-B-19]: Solve graphically the trigonometric equations of

the type: $\sin\theta = \frac{\theta}{2}$,
 $\cos\theta = \theta$, $\tan\theta = 2\theta$
 where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

- [SLO: M-12-B -20]: Use the periods of trigonometric functions to find the general solution of

		<p>periodicity, even odd functions, extreme values to real world problems such as (distance, elevation, and direction of tall structures, navigation and mapping, lengths of irregular shapes, graphs to visualize and predict patterns in data, frequency and periodic length of Ferris wheel, forces on a see-saw or lever, the ideal angle for solar panel placement).</p>	<p>the trigonometric equations.</p> <ul style="list-style-type: none"> • [SLO: M-12 -B -21] Apply concepts of inverse trigonometric functions to real lifeworld problems (such as mechanical engineering, architecture to find the height of the building, angle of elevation and depression, identifying the angle of bridges to build scale models).
	<p>Chords and Arcs of a Circle</p> <ul style="list-style-type: none"> • [SLO: M-10-B-15]: Solve problems by using the property of a circle: One and only one circle can pass 		<p>CONICS</p> <ul style="list-style-type: none"> • [SLO: M-12-B -22]: Demonstrate conics and members of its family i.e. circle, parabola, ellipse and hyperbola.

	<p>through three non-collinear points.</p> <ul style="list-style-type: none"> • [SLO: M-10-B-16]: Solve problems by using the property of circle: A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord. • [SLO: M-10-B-17]: Solve problems by using the property of a circle: Perpendicular from the centre of a circle on a chord bisects it. • [SLO: M-10-B-18]: Solve problems by using the property of circle: If two chords of a circle are congruent then they 		<p>Circle, Equation of a Circle</p> <ul style="list-style-type: none"> • [SLO: M-12-B -23]: Derive and apply equation of a circle in standard form i.e. $(\bar{x} - h)^2 + (\bar{y} - \bar{k})^2 = \bar{r}^2$ • [SLO: M-12-B -24]: Find the equation of a circle passing through: three non collinear points, two points and having its centre on a given line, two points and equation of tangent at one of these points is known, two points and touching a given line. <p>Tangent and Normal</p> <ul style="list-style-type: none"> • [SLO: M-12-B -25]: Find the condition when:
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	<p>will be equidistant from the centre</p> <ul style="list-style-type: none"> • [SLO: M-10-B-19]: Solve problems by using the property of a circle: Two chords of a circle which are equidistant from the centre are congruent. • [SLO: M-10-B-20]: Solve problems by using the property of circle: If two arcs of a circle (or of congruent circles) are congruent then the corresponding chords are equal. • [SLO: M-10-B-21]: Solve problems by using the property of circle: If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs 		<p>a line intersects the circle. a line touches the circle.</p> <ul style="list-style-type: none"> • [SLO: M-12-B -26]: Find the equation of a tangent: to a circle in slope form and a normal to a circle at a point. • [SLO: M-12-B -27]: Find the length of tangent to a circle from a given external point. <p>Parabola</p> <ul style="list-style-type: none"> • [SLO: M-12-B -28]: Derive and apply the standard equation of a parabola. • [SLO: M-12-B -29]: sketch their graphs and find their elements.
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	<p>(minor, major or semi-circular) are congruent.</p> <ul style="list-style-type: none"> • [SLO: M-10-B-22]: Solve problems by using the property of circle: Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres). • [SLO: M-10-B-23]: Solve problems by using the property of circle: If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding centres) are equal, the chords are equal. • [SLO: M-10-B-24]: Apply concepts of 		<ul style="list-style-type: none"> • [SLO: M-12-B -30]: Find the equation of a parabola with the following given elements: focus and vertex, focus and directrix, vertex and directrix. <p>Equations of Tangent and Normal</p> <ul style="list-style-type: none"> • [SLO: M-12-B-31]: Find the condition when a line is tangent to a parabola at a point and hence write the equation of a tangent line in slope form. • [SLO: M-12-B-32]: Find the equation of a tangent and a normal to a parabola at a point. <p>Ellipse</p>
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chords and arcs of a circle to real life world problems (such as decorative features, rainbow, bridges, roller coaster track).

Tangent and Angles of a Circle

- [SLO: M-10-B-25]: Solve problems by using the property of circle: If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.
- [SLO: M-10-B-26]: Solve problems by using the property of a circle: The tangent to a circle and the radial segment joining the point of contact

- [SLO: M-12-B-33]: Derive and apply the standard form of equation of an ellipse and identify its elements.
- [SLO: M-12-B-34]: Convert a given equation to the standard form of equation of an ellipse, find its elements and draw the graph.

Equations of Tangent and Normal

- [SLO: M-12-B -35]: Find points of intersection of an ellipse with a line including the condition of tangency.
- [SLO: M-12-B-36]: Find the equation of

	<p>and the centre are perpendicular to each other.</p> <ul style="list-style-type: none"> • [SLO: M-10-B-27]: Solve problems by using the property of circle: The two tangents drawn to a circle from a point outside it, are equal in length. • [SLO: M-10-B-28]: Solve problems by using the property of a circle: If two circles touch externally or internally, the distance between their centres is respectively equal to the sum or difference of their radii. • [SLO: M-10-B-29]: Solve problems by using the property of circle: The measure of 		<p>a tangent to an ellipse in slope form.</p> <ul style="list-style-type: none"> • [SLO: M-12-B -37]: Find the equation of a tangent and a normal to an ellipse at a point. <p>Standard Form of Equation of Hyperbola</p> <ul style="list-style-type: none"> • [SLO: M-12-B -38]: Derive and apply the standard form of equation of a hyperbola and identify its elements. • [SLO: M-12-B -39]: Find the equation of a hyperbola with the following given elements: transverse and conjugate axes with centre at origin, two points, eccentricity, latera recta and transverse
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	<p>a central angle of a minor arc of a circle is double that of the angle subtended by the corresponding major arc.</p> <ul style="list-style-type: none"> • [SLO: M-10-B-30]: Solve problems by using the property of a circle: Any two angles in the same segment of a circle are equal. • [SLO: M-10-B-31]: Solve problems by using the property of circle: The angle in a semi-circle is a right angle, in a segment greater than a semi-circle is less than a right angle, in a segment less than a semi-circle is greater than a right angle. 		<p>axes, focus, eccentricity and centre, focus, centre and directrix.</p> <p>Equations of Tangent and Normal</p> <ul style="list-style-type: none"> • [SLO: M-12-B-40]: Find points of intersection of hyperbola with a line including the condition of tangency. • [SLO: M-12-B-41]: Find the equation of a tangent to a hyperbola in slope form. • [SLO: M-12-B-42]: Find the equation of a tangent and a normal to a hyperbola at a point. • [SLO:M-12 -B-43]: Apply concepts of
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	<ul style="list-style-type: none"> • [SLO: M-10-B-32]: Solve problems by using the property of circle: The opposite angles of any quadrilateral inscribed in a circle are supplementary. • [SLO: M-10-B-33]: Apply concepts of tangents and angles of a circle to real life world problems (such as architecture, monuments, pyramids) 		<p>conics to real life world problems (such as suspension and reflection problems related to parabola, Satellite system, elliptic movement of electrons in the atom around the nucleus, radio system use as hyperbolic functions, flashlights, conics in architecture).</p>
<p>Construction of Triangle</p> <ul style="list-style-type: none"> • [SLO: M-09-B-28]: Construct a triangle having given two sides and the included angle. • [SLO: M-09-B-29]: Construct a triangle 	<p>Practical Geometry of Circles</p> <ul style="list-style-type: none"> • [SLO: M-10-B-34]: Locate the centre of a given circle. • [SLO: M-10-B-35]: Draw a circle passing 		

<p>having given one side and two of the angles.</p> <ul style="list-style-type: none"> • [SLO: M-09-B-30]: Construct a triangle having given two of its sides and the angle opposite to one of them (with all the three possibilities). • [SLO: M-09-B-31]: Draw angle bisectors, perpendicular bisectors, medians, altitudes of a given triangle and verify their concurrency. 	<p>through three given noncollinear points.</p> <ul style="list-style-type: none"> • [SLO: M-10-B-36]: Complete the circle: <ul style="list-style-type: none"> • by finding the centre, • without finding the centre, when a part of its circumference is given. <p>Tangent to the Circle</p> <ul style="list-style-type: none"> • [SLO: M-10-B-37]: Draw a tangent to a given arc, without using the centre, through a given point P when P is <ul style="list-style-type: none"> • the middle point of the arc, • at the end of the arc, • outside the arc. • [SLO: M-10-B-38]: Draw a tangent to a given circle from a point P when P lies 		
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	<ul style="list-style-type: none">• on the circumference• outside the circle• [SLO: M-10-B-39]: Draw two tangents to a circle meeting each other at a given angle.• [SLO: M-10-B-40]: Apply concepts of practical geometry of a circle to real life world problems (such as athletic tracks, recreational parks, ferris wheels, mechanical machines)		
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RATIONALE

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise. Statistics is used to describe and analyse phenomena involving uncertainty and variation. For these reasons this domain provides a foundation for further studies in disciplines in which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. In summary, the curriculum is designed for students whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level. In the 2006 National curriculum, the percentage of statistical concepts as compared to O-level and A-level is not appropriate therefore concepts of normal distribution are added to the SLOs of Grade XII and Permutation/Combination are shifted to grade XI.

Domain C: Information Handling

Standard-3 The students will be able to collect, organize, analyze, display and interpret data/ information.

Grade 9	Grade 10	Grade 11	Grade 12
Benchmark I: Students will be able to find measures of central tendency and dispersion to draw conclusion, construct and interpret cumulative frequency curve, measure correlation using scatter diagram, Benchmark II: Students will be able to predict the outcomes of single and combined events using diagrams,		Benchmark I: Benchmark II: Students will be able to solve problems involving permutations and combinations	

find probability and recognize probabilities of compound events.

Student Learning Outcomes

Frequency Distribution

- [SLO: M-09-C -01]: Construct a grouped frequency table, histogram (with unequal class intervals) and frequency polygon

Measure of Central Tendency

- [SLO: M-09-C -02]: Calculate the mean modal class and median of a grouped frequency distribution
- [SLO: M-09-C -03]: Solve real life situations involving mean, weighted mean,

Cumulative Frequency Distribution and measures of dispersion

- [SLO: M-10-C-01]: Construct cumulative frequency table, cumulative frequency polygon or Ogive
- [SLO: M-10-C-02]: Interpret the median, quartiles, deciles, percentiles, and inter quartile range from cumulative frequency curve.
- [SLO: M-10-C-03]: Interpret and analyse box and whisker plots
Correlation

median, and mode for given data (such as allocation of funds in different projects, forecasting future demographics, marketing, forecasting government budgets).

- [SLO: M-10-C-04]: Construct and interpret data from scatter diagrams and also draw lines of best fit
- [SLO: M-10-C-05]: Measure correlation using scatter diagram

Measure of Dispersion

- [SLO: M-10-C-06]: Calculate the range, standard deviation and variance for grouped data.
- [SLO: M-10-C-07]: Use the mean and standard deviation to compare two sets of data
- [SLO: M-10-C-08]: Solve real life situations involving variance, and standard deviation for grouped data

	<ul style="list-style-type: none"> • [SLO: M-10-C-09]: Apply concepts from measures of dispersion to solve real life situations (such as determining the consistency of data, checking variability in forecasting, manufacturing, finance, economics). 		
<p>Probability</p> <ul style="list-style-type: none"> • [SLO: M-09-C -04]: Calculate the probability of a single event and the probability of event not occurring • [SLO: M-09-C -05]: Solve real life problems involving probability 	<p>Probability of Combined Events</p> <ul style="list-style-type: none"> • [SLO: M-10-C-10]: Calculate the probability of combined events using, where appropriate: sample space diagrams, possibility diagram, tree diagrams, Venn diagrams. 	<p>Permutation and Combination</p> <ul style="list-style-type: none"> • [SLO: M-11-C-01]: Explain and solve problems that involve the fundamental counting principle. • [SLO: M-11-C-02]: Explain and Solve problems that involve permutations. • [SLO: M-11-C-03]: Explain and Solve 	

Relative and expected frequencies

- [SLO: M-09-C -06]: Calculate relative frequency as an estimate of probability.
- [SLO: M-09-C -07]: Calculate expected frequencies.
- [SLO: M-09-C -08]: Solve real life problems involving relative and expected frequencies.

- [SLO: M-10-C-11]: Apply addition law of probability to solve problems involving mutually exclusive events (such as left and right hand turns, tossing a coin, even and odd numbers on a die, winning and losing a game)
- [SLO: M-10-C-12]: Apply the Multiplication law of probability to solve problems involving independent and dependent events (trading, flipping a coin, such as 2 cards being drawn 1 by 1 with replacement and without replacement etc.)

problems that involve combinations.

- [SLO: M-11-C -04]: Apply the concepts of permutation and combination to real world problems such as (cryptography, estimating the odds of winning a lottery, calculating the number of possible DNA sequences or protein structures, choosing different sets of songs for certain occasions)



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